Closing today: HW_1A,1B,1C
Closing next wed: HW_2A,2B,2C
Office Hours: 1:30-3:00pm in Smith 309

Quick review:

Def'n: The "signed" area between $f(x)$ and the x-axis from $x = a$ to $x = b$ is the *definite integral*:

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x,$$

where $\Delta x = \frac{b-a}{n}$ and $x_i = a + i\Delta x$

FTOC(1): Areas are antiderivatives!

$$\frac{d}{dx}\left(\int_a^x f(t)dt\right) = f(x)$$

FTOC(2): If F(x) is <u>any</u> antideriv. of $f(x)$,

$$\int_a^b f(x)dx = F(b) - F(a)$$

Entry Task: Evaluate

$$\int_0^4 e^x + \sqrt{x^3}\,dx$$

$$\int_3^6 \frac{4}{x} - \frac{2}{x^2}\,dx$$

5.4 The Indefinite Integral and Net/Total Change

Def'n: The **indefinite integral** of $f(x)$ is defined to be the general antiderivative of $f(x)$. And we write

$$\int f(x)dx = F(x) + C,$$

where $F(x)$ is any antiderivative of $f(x)$.

Net Change and Total Change

Assume an object is moving along a straight line (up/down or left/right).

Let

$s(t)$ = 'location at time *t*'

$v(t)$ = 'velocity at time t'

pos. $v(t)$ means moving up/right

neg. $v(t)$ means moving down/left

The FTOC (part 2) says

$$\int_a^b v(t)dt = s(b) - s(a)$$

i.e.

'integral of velocity'= '**net change** in dist'

We also call this the *displacement*.

In general, the FTOC(2) says the **net change** in $f(x)$ from $x = a$ to $x = b$ is the integral of its rates.
That is:

$$\int_a^b f'(t)dt = f(b) - f(a)$$

We define **total change** in dist. by

$$\int_a^b |v(t)|dt$$

which we compute by

1. Solving $v(t) = 0$ for t.

2. Splitting up the integral at these t values, dropping the absolute value and integrating separately.

3. Adding together as positive numbers.

5.5 Substitution - *Motivation*:

1. Find the following derivatives

Function	Derivative?
$\cos(x^2)$	
$\sin(x^4)$	
$e^{\tan(x)}$	
$(\ln(x))^3$	
$\ln(x^4 + 1)$	

2. Rewrite each as integrals:

$$\int \qquad dx = \cos(x^2) + C$$

$$\int \qquad dx = \sin(x^4) + C$$

$$\int \qquad dx = e^{\tan(x)} + C$$

$$\int \qquad dx = (\ln(x))^3 + C$$

$$\int \qquad dx = \ln(x^4 + 1) + C$$

3. Guess and check the answer to:

$$\int 7x^6 \sin(x^7)\, dx =$$

Observations:

1. We are reversing the "chain rule".
2. In each case, we see
   "inside" = function inside another
   "outside" = derivative of inside

To help us mechanically see these connections, we use what we call:

The Substitution Rule:
If we write u = g(x) and du = g'(x) dx, then

$$\int f(g(x))g'(x)dx = \int f(u)du$$